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Evolution Strategy Optimization for Adiabatic Pulses in MRI

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We propose a new type of adiabatic pulses for uniform inversion of the magnetization in magnetic resonance imaging. We produced these pulses with an evolution strategy optimization, by which the search of the "best solution" has been made more efficient than by deterministic algorithms. The pulse parametrization takes into account an "offset-independent adiabaticity condition," which guarantees insensitivity to RF inhomogeneities. The RF pulse power (both peak and mean) contributes to the cost to be minimized, as well as the error function does: in this way we obtain solutions that require lower energy than the well-known hyperbolic-secant pulse, with no loss of quality in the response profile. © 1999 Academic Press

Key Words: **adiabatic pulses; optimization; evolution strategy; offset-independent adiabaticity; power reduction.**

1. INTRODUCTION

A major problem of selective excitation in magnetic resonance imaging (MRI) is the inhomogeneity of the RF field; the fields of surface coils vary substantially over the region of interest (ROI) while the homogeneity of "volume coils" is often poor for a number of reasons: eddy currents, RF penetration, and tradeoffs among homogeneity, filling factor, and sensitivity. The problem is particularly severe in high-field imaging: our new 200-MHz birdcage coil has over a 50% variation over its specified ROI, a sphere of 12 cm in diameter. Since it is impossible, or inconvenient, to eliminate many sources of RF inhomogeneity, we should devise excitations which, to some degree, are insensitive to the RF strength.

An early solution was offered by the composite pulse sequences (*1*), which were obtained by modulating the phase of an excitation of constant amplitude. They are no longer popular, since their selectivity is not exceptional and power deposition is high, because they work best with very large tip angles. Today, the RF inhomogeneity problem is mostly handled with the so-called adiabatic pulses, which efficiently achieve nearly perfect inversion of the magnetization over the specified frequency band when the RF intensity is above a threshold value.

The general idea of an adiabatic pulse is to move the effective RF field, seen by a spin, from the "up" to the "down" direction by sweeping the frequency of the excitation; this sweep should be slow enough to satisfy the adiabatic theorem (*2*), hence the name. The best known adiabatic pulse is the so-called hyperbolic-secant pulse (*3*), with sech/tan h as the functions describing the amplitude/frequency sweep. Subsequently, many other modulation pairs have been proposed: constant/linear (*4*) (CHIRP pulses), (1-sin^n)/linear (WURST (*5*) pulses), constant/tan (*3*), cos/sin (*6*), tanh/tan (*7*); furthermore other function pairs have been generated through nonlinear transformations (*8*). The main applications of adiabatic pulses are broadband decoupling, solvent suppression, and imaging, particularly with inversion-recovery experiments (e.g., T_1 mapping); adiabatic pulses also perform other types of spin transformation (*6, 7*).

As it has been found that the sinc pulse is not the unique, nor the "best" solution of the selective excitation problem, similarly, it has been discovered that there are many "good" adiabatic inversion pulses, and they can be identified through numerical optimization. The NOM (numerically optimized modulation) pulses were early proposed by Ugurbil *et al.* (*9*) to reduce the RF sensitivity over a specified bandwidth and B_1 intensity range. However, it seems reasonable to introduce a constraint which limits the search to pulses satisfying the adiabatic theorem. In this way, we avoid the time-consuming task of testing the compliance of the "solution" with the specified B_1 intensity, as long as we have RF fields higher than envisioned by the solution. For example, Rosenfeld *et al.* (*10–12*) optimize over the sweep rate of the effective-field trajectory. The constraint of Shen (*13*) is that the adiabaticity factor is maximum at resonance, while Kupce and Freeman (*14*) and Tannús and Garwood (*15*) require an "offset-independent adiabaticity factor." The latter constraint translates into a prescription for the amplitude modulation alone, since the frequency modulation can be derived from the amplitude function and the offset-independent adiabaticity condition.

Our approach is different from the ones quoted above in that it searches among pulses with offset-independent adiabaticity factor *and* minimum power: doing so, we have found that our solutions yield the threshold B_1 values for the most efficient pulse. The other ingredients of the method we propose are a robust stochastic optimizer, successfully used to obtain other types of selective pulses (*16*), and the choice of the "function spaces" where the search is performed. We have tried several approaches, including linear combinations of "promising"

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functions, and we have found the best results with functions which generalize the idea behind the hyperbolic secant (HS) pulse.

Our adiabatic pulses have performances similar, or distinctly better, than those published so far; they are usually obtained in few minutes of computation (with a Pentium PC). We found that there are many solutions, i.e., substantially different pulses which have essentially the same performances. Our approach allows a compromise between power, pulse time, and accuracy requirements, and we believe that it embodies, and generalizes, the major part of the numerical approaches presented so far to create adiabatic pulses. In Section 2, we present the empirical amplitude functions, define the "cost" function, and summarize the optimization technique. Results of the simulations are commented on in Section 3 while experimental results are given in Section 4.

2. METHODS

We consider the class of offset-independent adiabaticity (OIA) pulses described in detail elsewhere (*14*, *15*). Following Baum *et al.* (*3*), we define the adiabaticity factor $Q(t)$ as

$$Q(t) = \frac{\omega_{\text{eff}}(t)}{|d\theta/dt|}. \quad [1]$$

The OIA pulses are obtained by requiring that the "adiabaticity factor in-resonance," Q_0, is constant, i.e., independent of the offset. This induces an integral relation between the frequency modulation $\Delta f(t)$ and the amplitude modulation $\omega_1(t)$,

$$\Delta f(t) = \frac{1}{Q_0}\int_0^t [\omega_1(t')]^2 dt'. \quad [2]$$

In the frame rotating at the carrier frequency ω, the phase of modulation $\phi(t)$ is

$$\phi(t) = \int_0^t \Delta f(t')dt'. \quad [3]$$

Condition [2] characterizes the OIA pulses, which have the following properties:

(i) they are adiabatic by design;

(ii) the phase modulation is simply related to the amplitude modulation;

(iii) as shown by Tannús and Garwood (*15*), the power is uniformly released over all the entire inversion band, a desirable feature;

(iv) the OIA constraint is not too rigid; as we will show, these pulses perform at least as well as adiabatic pulses satisfying other conditions.

Our method involves a parametrization of the amplitude function $\omega_1(t)$, from which the frequency and phase functions are obtained via numerical integration of Eqs. [2] and [3]. We use frequencies and time normalized with the pulse duration T_P,

$$w = \omega T_P, \quad \tau = t/T_P. \quad [4]$$

We want an amplitude function $w_1(\tau)$ symmetric and regular which increases in the first half of the pulse and decreases in the second half, as the hyperbolic secant does. We consider first the pulse

$$w_1(\tau) = 2\pi w_0 \operatorname{sech}[2\pi\beta(\tau - 0.5)]. \quad [5]$$

This is simply a HS pulse, but since we will optimize over the truncation factor β and relative amplitude w_0, we will indicate the so-found solutions by another label, SC, to stress that no condition has been explicitly imposed upon β and w_0, as it is the case of most HS pulses. Furthermore, we will consider the following two functions:

(i) stretched hyperbolic secant "SQ",

$$w_1(\tau) = 2\pi w_0 \frac{1}{\pi}\{\text{arc tan}[b\ \tan(\pi\{\operatorname{sech}[2\pi\beta \times (\tau - 0.5)]\}^a - \pi/2) + c] + \pi/2\} \quad [6a]$$

$$\Delta F(\tau) = \frac{1}{Q_0}\int [w_1(\tau)]^2 d\tau \quad [6b]$$

$$\phi(\tau) = \int \Delta F(\tau)d\tau \quad [6c]$$

with six parameters, namely, the adiabaticity factor Q_0, the relative amplitude w_0, the hyperbolic secant truncation-level β, the stretching coefficients a, b, and c;

(ii) stretched cosine (See also Eqs. [6b, and 6c]) "CQ",

$$w_1(\tau) = 2\pi w_0 \frac{1}{\pi}\{\text{arc tan}[b\ \tan(\pi\{\cos[\pi(\tau - 0.5)]\}^a - \pi/2) + c] + \pi/2\} \quad [7]$$

with five parameters, since the truncation level β is absent in the cosine function, which is always zero at the beginning and at the end of the pulse. When $a \ll 1$ we reproduce the wide amplitude modulation of WURST pulses proposed by other authors.

TABLE 1
Comparison of Adiabatic Pulses

Pulse	N points	N parameters	Selectivity	Q_0	P_{pk}	P_m	RMS error (%)
SC50	256	2	0.02	4.95	2.95	0.62	7.4
SQ50	256	6	0.02	4.7	2.92	0.61	6.8
CQ50	256	5	0.02	3.49	1.24	0.45	8.1
SC40	128	2	0.025	4.89	3.52	0.77	8.1
SQ40	128	6	0.025	4.53	3.29	0.75	7.6
CQ40	128	5	0.025	3.49	1.58	0.57	8.7
SC30	128	2	0.0333	4.41	4.05	0.92	9.2
SQ30	128	6	0.0333	3.84	3.67	0.85	8.6
CQ30	128	5	0.0333	3.75	2.2	0.82	9.6
SC20	128	2	0.05	3.79	4.88	1.19	10.9
SQ20	128	6	0.05	3.02	3.85	1	10.7
CQ20	128	5	0.05	3.02	2.72	0.99	11.2
SC15	128	2	0.0667	3.71	6.12	1.56	12.3
SQ15	128	6	0.0667	2.77	4.59	1.22	12.1
CQ15	128	5	0.0667	3.01	3.84	1.31	12.7
SC10	128	2	0.1	3.08	6.9	1.93	14.5
SQ10	128	6	0.1	2.13	4.78	1.38	14.3
CQ10	128	5	0.1	2.52	4.99	1.64	14.4

The target functions of the magnetization components are

$$M_x^{\mathrm{T}}(\Delta w) = 0 \quad [8a]$$

$$M_y^{\mathrm{T}}(\Delta w) = 0 \quad [8b]$$

$$M_z^{\mathrm{T}}(\Delta w) = -1 \quad [8c]$$

when $|\Delta w| \leq \pi/\sigma$, while

$$M_x^{\mathrm{T}}(\Delta w) = 0 \quad [9a]$$

$$M_y^{\mathrm{T}}(\Delta w) = 0 \quad [9b]$$

$$M_z^{\mathrm{T}}(\Delta w) = 1 \quad [9c]$$

when $\pi/\sigma < |\Delta w| \leq w_{\max}$ and $w_{\max}$ is the sampling limit. As previously done (*16*), the selectivity σ is defined as $(BW \cdot T_{\mathrm{P}})^{-1}$. The cost function that is to be minimized measures both the "distance" from the target (error function) and the power of the pulse

$$E = \sum_{\alpha} X_{\alpha}^2 + \lambda_1 p_{pk} + \lambda_2 p_m, \quad [10]$$

where $\alpha = x, y, z$ and we put

$$X_{\alpha}^2 = \frac{\sum_{k=1}^{N_{\mathrm{r}}} [M_{\alpha}(\Delta w_k) - M_{\alpha}^{\mathrm{T}}(\Delta w_k)]^2}{N_{\mathrm{r}}} \quad [11]$$

$$p_{pk} = \sigma^2\, \mathrm{MAX}_{\{n\}}[|w_1(\tau_n)|^2]; \quad [12]$$

$$p_m = \frac{\sigma^2}{N} \sum_{n=1}^{N} |w_1(\tau_n)|^2. \quad [13]$$

In these equations N_{r} indicates the number of sampled frequency point and N the number of sampled time points. The λ_1, λ_2 coefficients should be carefully set during the optimization of each pulse, according to the relative magnitudes of the response profile error, p_{pk} and p_m. The first term in Eq. [10] is the error function, i.e., the sum of the mean square errors of the three magnetization components $X_x^2 + X_y^2 + X_z^2$; it measures the discrepancies between actual and ideal magnetization profiles. The root square of the error function is the RMS error which will be given in percentages in Table 1.

To find the minimum of the cost function in the parameter space, we use a stochastic, evolution strategy algorithm (*16*), in which a "parent" solution p generates a "son" x with a probability density

$$\rho(x, p, d) = \exp\left[-\sum_i \left(\frac{x_i - p_i}{d_i}\right)^2\right]; \quad [14]$$

the dispersion d is increased when lots of improvements arise (far from a solution) and decreased otherwise (near a solution).

3. RESULTS AND DISCUSSION

We established a set of selectivity values, for which we want to compare the performance of the hyperbolic-secant pulse to the new solutions. The results are given in Table 1. The pulse name consists of a label and two digits. As we said, the label SC indicates hyperbolic-secant pulses, while SQ is used for OIA pulses optimized stretching the secant shape and CQ for those optimized stretching the cosine; the two digits are the

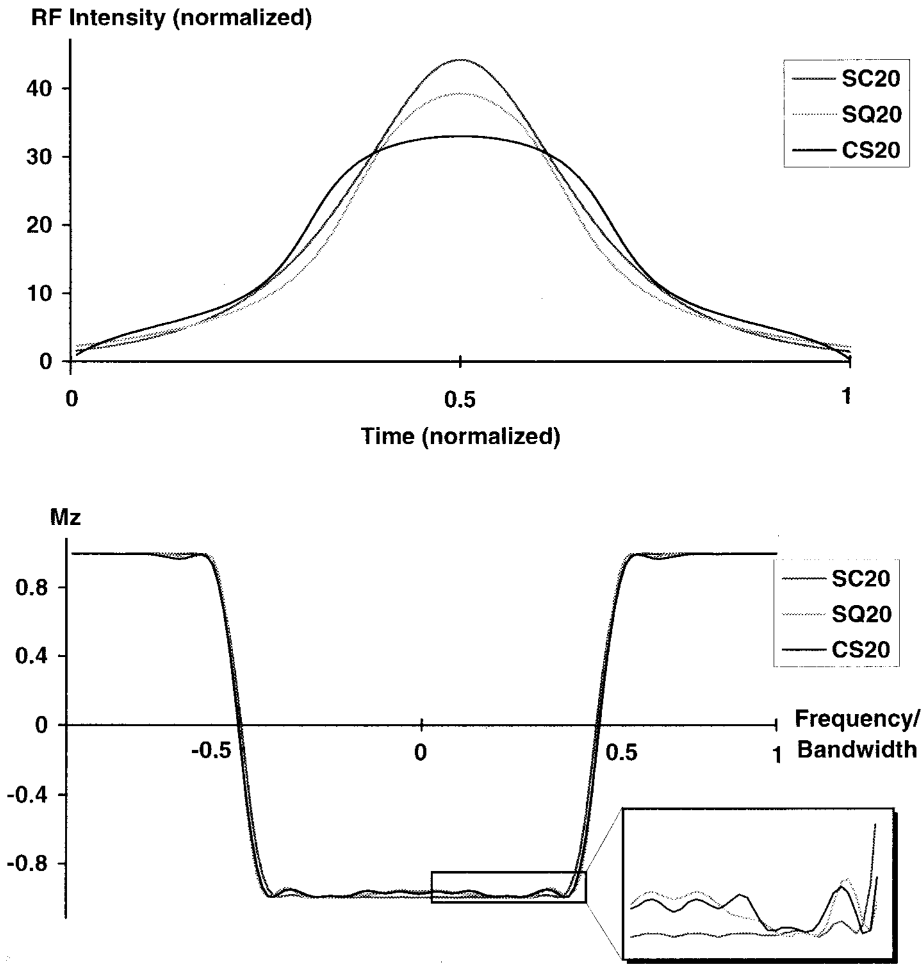

FIG. 1. Comparison of adiabatic pulses with $\sigma = 1/20$.

reciprocal of the selectivity ($BW \cdot T_P$). Notice that the SQ have a response slightly better and a power slightly lower than the SC; the CQ have a profile slightly worse, but show a considerable power reduction relative to the other pulses. As an example, we illustrate in Fig. 1 the comparison between the three pulses with selectivity 1/20 and in Fig. 2 the solutions with $\sigma = 1/40$.

We can easily recover with our strategy results achieved by other authors. Machann *et al.* (*18*) optimized the truncation factor β for a particular choice of the selectivity, i.e., $\sigma = 1/10$; they considered OIA pulses obtained from amplitude functions with Lorentzian, Gaussian, and squared cosine shape. Our method is more general since the pulse amplitude too is subjected to optimization and, with our parametrization, we can easily recover their results as special cases. In the cited papers of Rosenfeld *et al.*, instead, the authors optimize the sweep rate of the effective field in the FM frame, for a fixed trajectory. They produced low-power adiabatic pulses for some values of the selectivity: with $\sigma = 1/72$ they had (*11*) $p_{pk} = 0.39$; with $\sigma = 1/60$ a value $p_{pk} = 0.57$ is obtained (*10*); with $\sigma = 1/45$, $p_{pk} = 0.49$ (with a poor inversion profile) (*12*); corresponding to $\sigma = 1/7$ they had (*10*) $p_{pk} = 9.87$. Our method is able to give solutions physically different but showing similar performances: with $\sigma = 1/72$ we have $p_{pk} = 0.37$; with $\sigma = 1/60$, $p_{pk} = 0.53$; with $\sigma = 1/45$, $p_{pk} = 0.47$ (where we need to lose something in response profile quality to gain in power reduction); with $\sigma = 1/7$, $p_{pk} = 10.7$. Moreover, we are not limited in any way by the choice of "magical" values for the selectivity, but we span a wide range, as it is shown in Table 1. In our website (*19*), we give the new adiabatic inversion pulses, for different values of the selectivity, in the standard SISCO-Varian format (ASCII files with 64, 128, or 256 couples of numbers representing phase and relative amplitude).

4. EXPERIMENTAL

We tested some of our stretched adiabatic pulses with a Sisco-Varian 4.7-T imager and compared them with the hyperbolic-secant pulses SC with the same selectivity. We used a water-filled cylinder as the phantom. We applied the sequence of Fig. 3, where

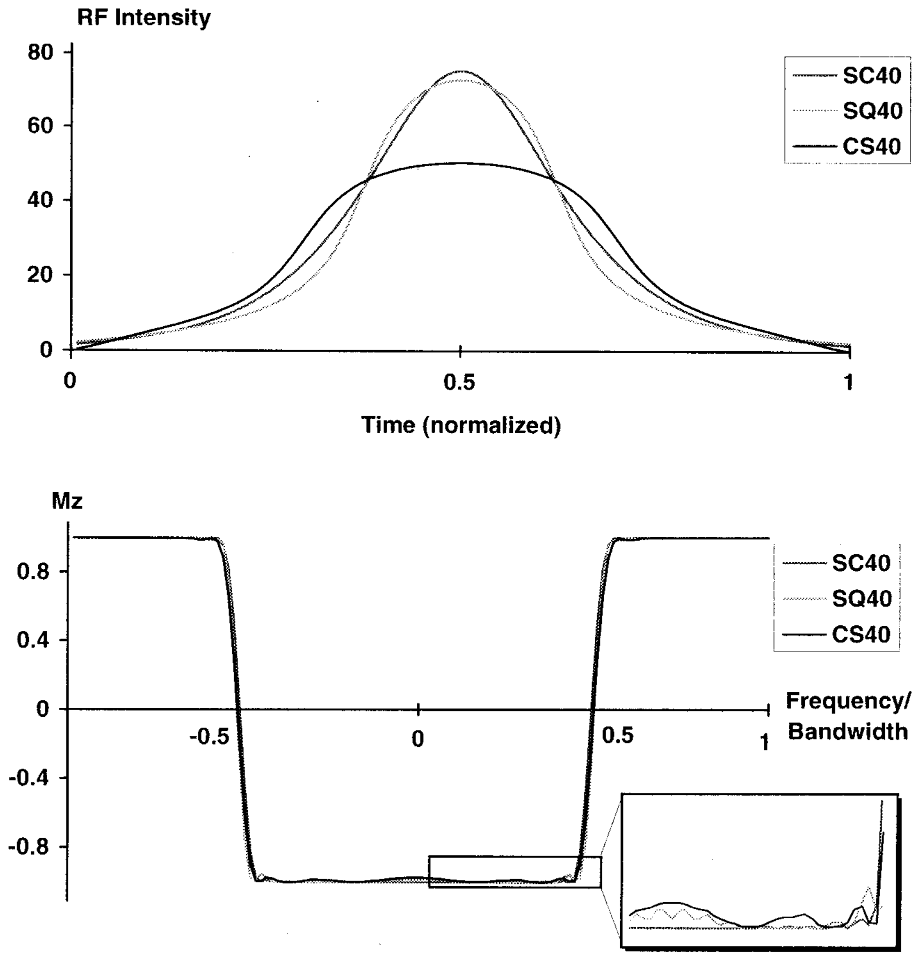

FIG. 2. Comparisons of adiabatic pulses with $\sigma = 1/40$.

the pulse to be tested is followed by the negative-phase 90° pulse L–3S4 (*16*, *19*), with a much larger bandwidth (12.5 kHz). At the end of the acquisition we make a Fourier transform and subtract from the absorption component the corresponding spectrum obtained without the first pulse. Figure 4 shows the results obtained with the pulses SC40 (a), SQ40 (b), and CQ40 (c). The power was measured as the reading, in dB, of the setting of the amplification applied to the first pulse and is reported in the figures. Relative to the SC pulse, note that the SQ improves the profile while the CQ needs less power.

5. CONCLUSIONS

We have demonstrated a simple, powerful method to make new adiabatic pulses which are comparable or better than the pulses so far proposed in the literature. In particular, we have shown that our pulses either improve the response profile and/or substantially reduce the power requirements, sometimes at the expense of a minor worsening of the profile. Our optimization strategy, applied to the offset-independent adiabaticity condition of Freeman *et al.*, is flexible enough to allow

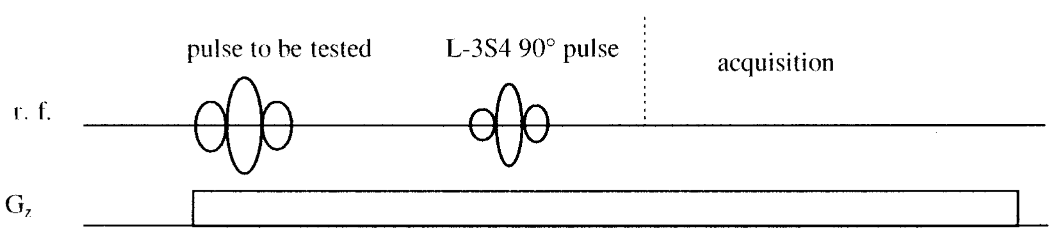

FIG. 3. Sequence for acquisition of 180° adiabatic pulses response.

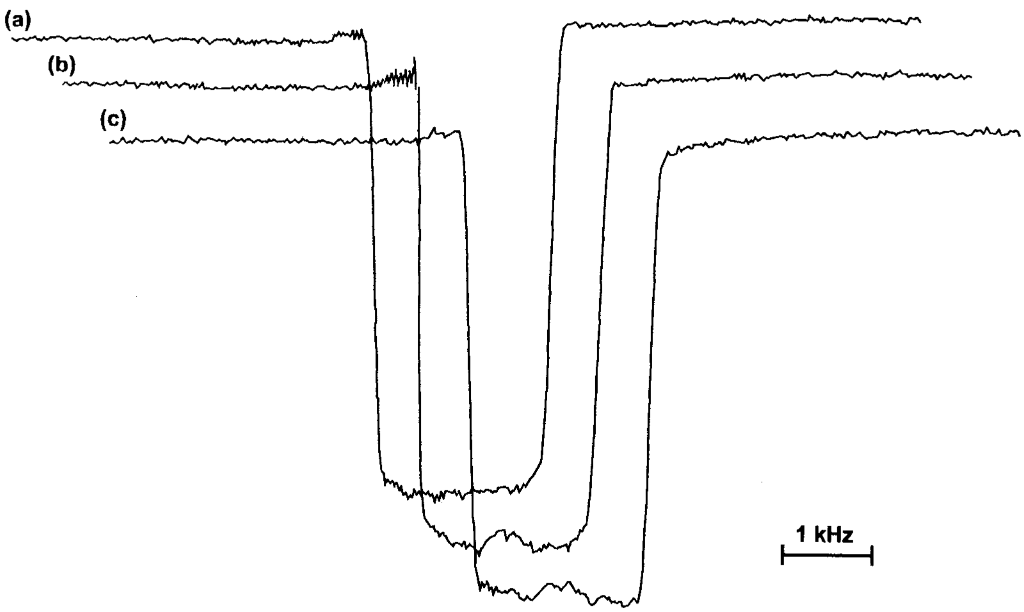

FIG. 4. Frequency response of (a) the pulse SC40 (+35 dB); (b) the pulse SQ40 (+34 dB); (c) the pulse CQ40 (+32 dB).

setting the compromise between power and accuracy, with a simple modification of our cost function.

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